logopt: log optimal portfolio

Marc Delvaux

November 8, 2010

Contents

1 Introduction

Portfolio selection is a vast subject, already well covered by existing R [3] packages. Package `logopt` is only concerned with a specific branch, portfolio selection algorithms that rely on a logarithmic utility function.

This vignette is still heavily under construction, all comments welcome.

2 Some history

The logarithmic utility function is tied to the concept of growth optimal portfolio, the portfolio that maximizes the growth rate of wealth. While maximizing the growth rate seems intuitively a good thing, it has also been criticized as leading to aggressive and thus volatile strategies.

Historically, the growth optimal approach is tied to information theory, starting with the initial analysis by Kelly [2].

Package `logopt` implements a number of portfolio selection algorithms that have been proposed in the literature and rely on the logarithmic utility function. This vignette shows how to reproduce classical results using the functions provided in `logopt` and in general shows how R coupled with modern hardware provides a wonderful tool for portfolio selection algorithms.

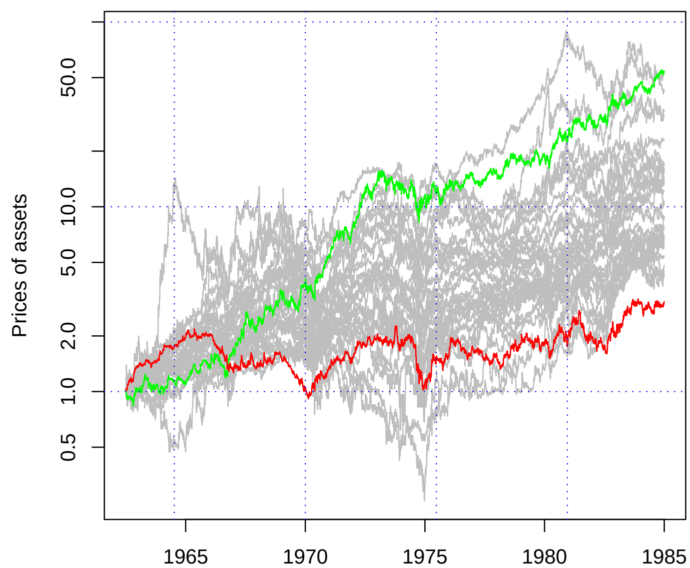
2.1 Cover Universal Portfolios

Thomas Cover article on "Universal Portfolios" [1] is quite important for two reasons:

- it provided an algorithm that asymptotically matched the growth rate of the best constant rebalanced portfolio for all possible sequences of stock prices
- it used a dataset of real stock price to show that 20 years was a long enough horizon to observe good results.

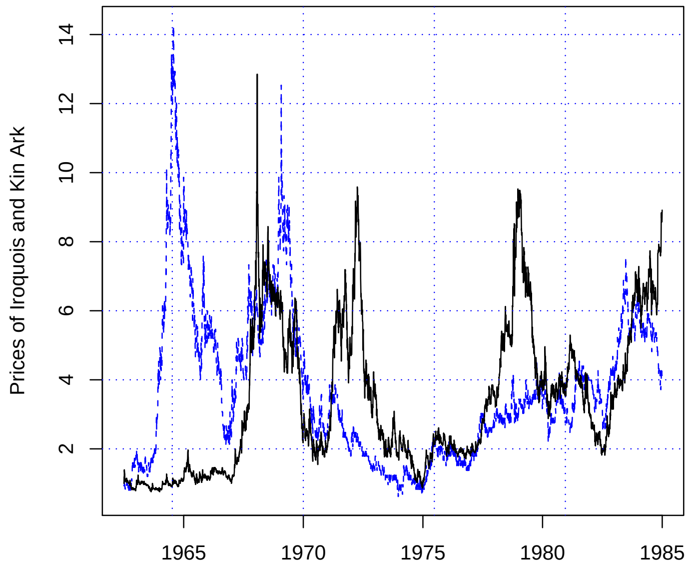
Package `logopt` includes the time series of prices relative used by Cover, retrieved from a website on log optimal portfolios. The next figure shows the time evolution of the different assets tracked. This shows the use of function `cumprod` to transform a time series of price relative into the time series of prices. From `logopt`, we also use `best.asset` and `worst.asset`, note that these two functions take the price relative as input but return a price series.

```
> require(logopt)
> data(nyse.cover.1962.1984)
> x <- nyse.cover.1962.1984
> prices <- cumprod(x)
> nDays <- dim(prices)[1]
> nAssets <- dim(prices)[2]
> plot(index(prices), prices[, 1], col = "gray", type = "l", ylim = range(prices),
+     ylab = "Prices of assets", log = "y")
> for (i in 2:nAssets) {
+     lines(index(prices), prices[, i], col = "gray", log = "y")
+ }
> lines(index(prices), best.asset(nyse.cover.1962.1984), col = "green",
+     log = "y")
> lines(index(prices), worst.asset(nyse.cover.1962.1984), col = "red",
+     log = "y")
> grid(col = "blue")
```



We can now reproduce the examples originally presented by Cover. First is figure 8.1 showing the time evolution of Iroquois and Kin Ark.

```
> x.ik <- nyse.cover.1962.1984[, c("kinar", "iroqu")]
> prices.ik <- cumprod(x.ik)
> plot(index(prices.ik), prices.ik[, 1], col = "blue", type = "l",
+     ylim = range(prices.ik), ylab = "Prices of Iroquois and Kin Ark",
+     lty = "dashed")
> lines(index(prices.ik), prices.ik[, 2], col = "black")
> grid(col = "blue")
```



Then figure 8.2, the terminal values of different CRP based on Iroquois and Kin Ark.

```
> alphas <- seq(0, 1, by = 0.05)
> crps <- alphas
> for (i in 1:length(crps)) {
+     alpha <- alphas[i]
+     crps[i] <- crp(x.ik, c(alpha, 1 - alpha))[nDays]
+ }
> plot(alphas, crps, col = "blue", type = "l", ylab = "Terminal values of CRP")
> abline(h = mean(crps), col = "green")
> grid(col = "blue")
```

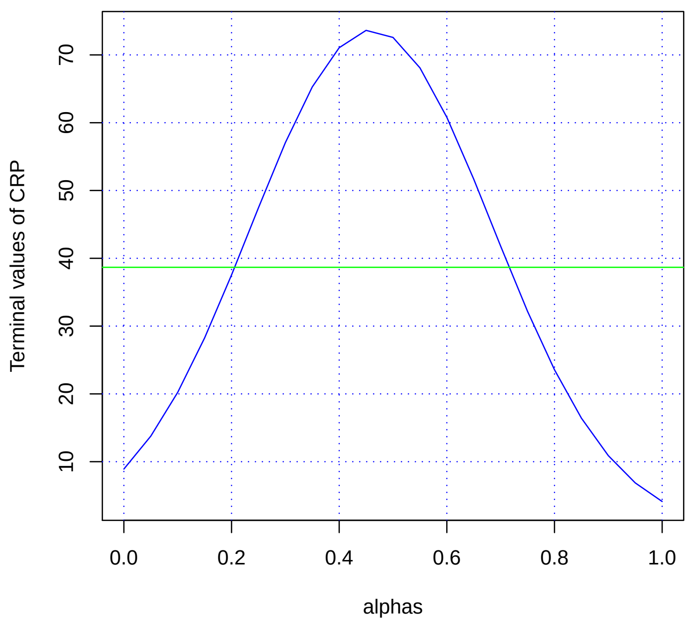


Table 8.1 contains the same information, Cover does indicate the best set of weights as (0.55, 0.45) with a terminal value of 73.619 based on the 0.05 grid. Function `bcrp.optim` can be used to get a more precise value and the associated terminal wealth.

```
> b.opt <- bcrp.optim(x.ik)
> crp.opt <- crp(x.ik, b.opt)
> print(sprintf("Best weights (%.3f,%.3f), terminal wealth %.3f",
+     b.opt[1], b.opt[2], crp.opt[nDays]))
```

```
[1] "Best weights (0.461,0.539), terminal wealth 73.701"
```

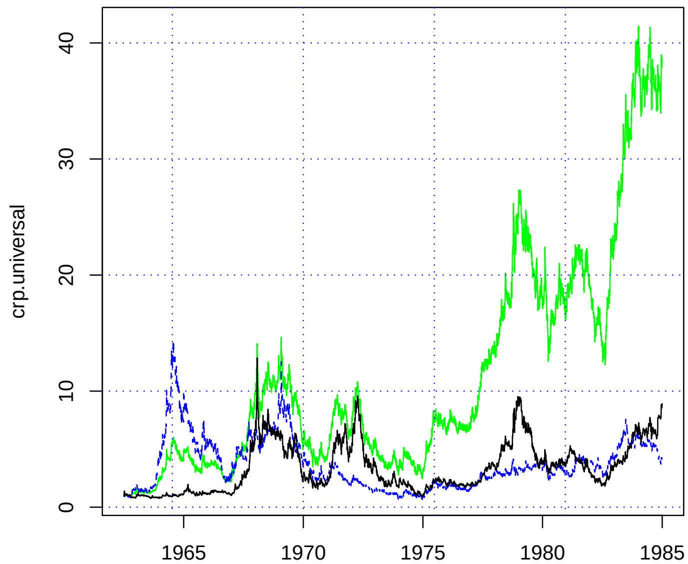
We can then reproduce Figure 8.3 with function `universal.cover` using the default grid method, similar to Cover's own approach. Because we use the same grid step, the terminal wealth of the universal portfolio matches exactly Cover's value of 38.6727.

```
> crp.universal <- universal.cover(x.ik, 20)
> print(sprintf("Terminal wealth of universal portfolio is %.4f",
+     crp.universal[nDays]))
```

```
[1] "Terminal wealth of universal portfolio is 38.6727"
```

```
> plot(index(prices.ik), crp.universal, col = "green", type = "l",
+     xlab = "Time series of wealth of universal portfolio and the underlying assets")
```

```
> lines(index(prices.ik), prices.ik[, 1], col = "blue", lty = "dashed")
> lines(index(prices.ik), prices.ik[, 2], col = "black")
> grid(col = "blue")
```



Time series of wealth of universal portfolio and the underlying assets

2.2 Side note, choice of Iroquois and Kin Ark as example

Cover mentions that Iroquois and Kin Ark were chosen as an example because of their volatility. What Cover does not mention (and possibly didn't remark) is that the pair formed by Kin Ark and Iroquois is very special. The code below calculates the optimal CRP for all possible pairs of assets in the dataset used by Cover.

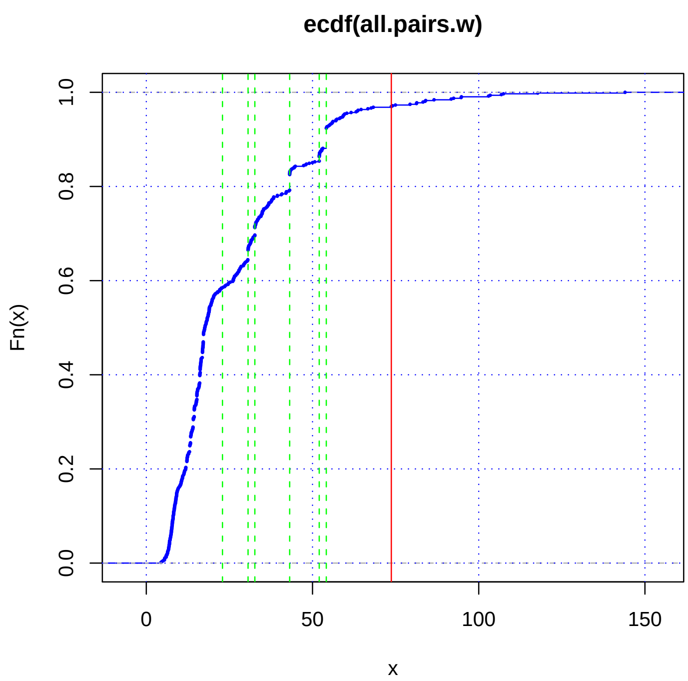
Caution, this code takes a long time to complete (about 30 minutes for a 2010 timeframe desktop).

```
> data(nyse.cover.1962.1984)
> x <- nyse.cover.1962.1984
> Indices <- t(combn(nAssets, 2))
> nPairs <- dim(Indices)[1]
> all.pairs.w <- Indices[, 1]
> all.pairs.b <- Indices
> for (i in 1:nPairs) {
+     print(colnames(x)[Indices[i, ]])
```

```
+     x.pair <- x[, Indices[i, ]]
+     bcrp.opt <- bcrp.optim(x.pair)
+     all.pairs.b[i, ] <- bcrp.opt
+     all.pairs.w[i] <- crp(x.pair, bcrp.opt)[nDays]
+ }
```

The pair formed by Kin Ark and Iroquois has a high but not extremal wealth, as shown below, the red vertical line identifies the terminal wealth of the optimal CRP for that pair, it in the tail but not the largest one. Note also the ecdf plot shows vertical gaps. The dashed vertical lines correspond to the 5 best terminal wealths for single assets, showing that the gaps correspond to case where the best CRP becomes simply the best asset in the pair.

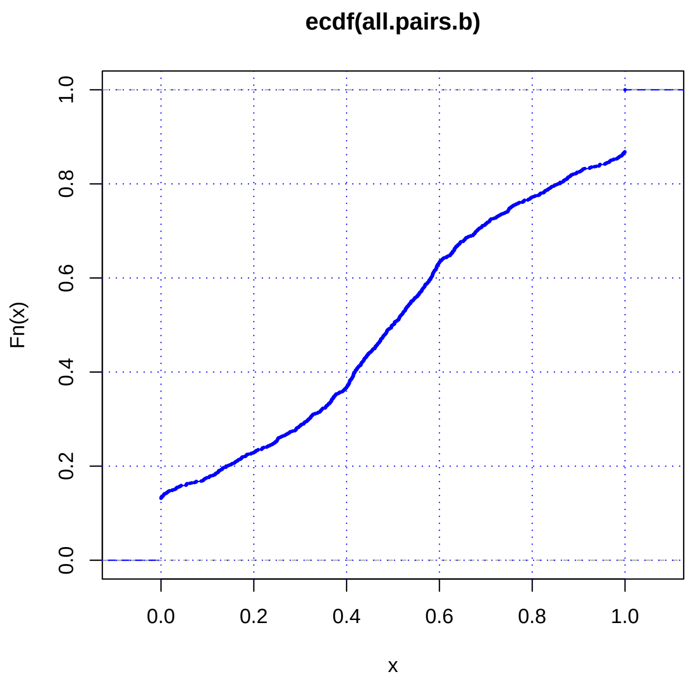
```
> plot(ecdf(all.pairs.w), col = "blue", cex = 0.25)
> grid(col = "blue")
> asset.wealths <- apply(x, 2, prod)
> abline(v = crp.opt[nDays], col = "red")
> for (i in 1:6) {
+     abline(v = asset.wealths[order(asset.wealths, decreasing = TRUE)[i]],
+         lty = "dashed", col = "green")
+ }
```



This can be further confirmed by examining the weights for the optimal portfolios, showing the vertical gap at 0 and 1. About 13% of the optimal CRP are made of a single asset from the underlying pair.

```
> plot(ecdf(all.pairs.b), col = "blue", cex = 0.25)
> grid(col = "blue")
> print(sum(all.pairs.b == 1)/prod(dim(all.pairs.b)))
```

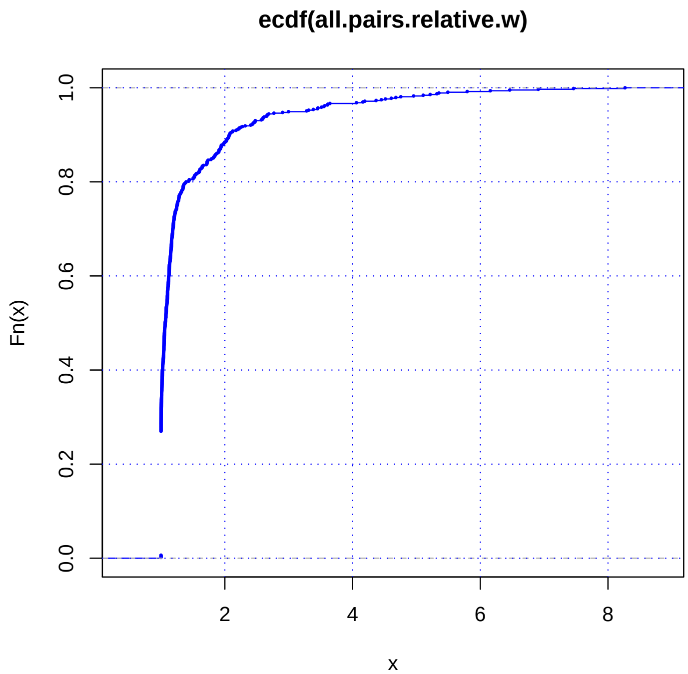
```
[1] 0.1317460
```



It is then natural to scale the terminal wealth of the optimal CRP for a given pair against the terminal wealth of the best asset in the pair. This shows the relative wealth improvement coming from the use of the optimal CRP. The corresponding plot shows that the relative wealth improvement is generally limited. The pair with the highest ratio between the optimal CRP and the best asset in the pair is formed by Iroquois and Kin Ark.

```
> all.pairs.relative.w <- all.pairs.w/apply(array(asset.wealths[Indices],
+     dim = dim(Indices)), 1, max)
> plot(ecdf(all.pairs.relative.w), col = "blue", cex = 0.25)
> grid(col = "blue")
> colnames(x)[Indices[which.max(all.pairs.relative.w), ]]
```

```
[1] "iroqu" "kinar"
```



3 Conclusion

Package `logopt` is a work in progress that hopefully will be found to be useful.

References

[1] Cover, T.~M. Universal portfolios. *Mathematical Finance 1*, 1 (1991), 1–29.

[2] Kelly, J. A new interpretation of information rate, Bell Syst. *Tech. J 35* (1956), 917–926.

[3] R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2010. ISBN 3-900051-07-0.