Asset selection with Local Search

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1 Introduction

We provide a code example for a simple asset selection problem; for more details, please see Gilli et˜al. [2011]. We start by attaching the package and fixing a seed.

```
> require("NMOF")
> set.seed(112233)
```

2 The problem

We wish to select between K_{inf} and K_{sup} out of n_{A} assets such that an equally-weighted portfolio of these assets has the lowest-possible variance. The formal model is:

$$\min_{w} w'\Sigma w \tag{1}$$

subject to the constraints

$$\begin{aligned} w_j = {}^1\!/_K \quad & \text{for } j \in J\,, \\ K_{\text{inf}} \leq K \leq & K_{\text{sup}}\,. \end{aligned}$$

The weights are stored in the vector w; the symbol J stands for the set of assets in the portfolio; and $K = \#\{J\}$ is the cardinality of this set, ie, the number of assets in the portfolio.

3 Setting up the algorithm

We start by attaching the package and creating random data. We simulate 500˜assets: each gets a random volatility between 20% and 40%, and all pairwise correlations are set to˜0.6.

```
> na <- 500L
> C <- array(0.6, dim = c(na, na)); diag(C) <- 1
> minVol <- 0.20; maxVol <- 0.40
> Vols <- (maxVol - minVol) * runif(na) + minVol
> Sigma <- outer(Vols, Vols) * C
```

The objective function.

```
> OF <- function(x, data) {
     xx <- as.logical(x)
     w <- x/sum(x)
     res <-  crossprod(w[xx], data$Sigma[xx, xx])
     res <- tcrossprod(w[xx], res)
     res
 }
```

. . . or even simpler:

```
> OF2 <- function(x, data) {
     xx <- as.logical(x); w <- 1/sum(x)
     res <- sum(w * w * data$Sigma[xx, xx])
     res
 }
```

The neighbourhood function.

```
> neighbour <- function(xc, data) {
     xn <- xc
     p <- sample.int(data$na, data$nn, replace = FALSE)
     xn[p] <- abs(xn[p] - 1L)
     ## reject infeasible solution
     if((sum(xn) > data$Ksup) || (sum(xn) < data$Kinf))
         xc else xn
 }
```

We collect all necessary information in the list `data`: the variance–corvariance matrix `Sigma`, the cardinality limits˜`Kinf` and `Ksup`, the total number of assets `na` (ie, the cardinality of the asset universe), and the parameter˜`nn`. This parameter controls the neighbourhood: it gives the number of assets that are to be changed when a new solution is computed.

```
> data <- list(Sigma = Sigma,
               Kinf = 30L,
               Ksup = 60L,
                 na = na,
                 nn = 1L)
```

4 Solving the model

As an initial solution we use a random portfolio.

```
> card0 <- sample(data$Kinf:data$Ksup, 1L, replace = FALSE)
> assets <- sample.int(na, card0, replace = FALSE)
> x0 <- numeric(na)
> x0[assets] <- 1L
```

With this implementation we need assume that `data$Ksup > data$Kinf`. (If `data$Ksup == data$Kinf`, then `sample` returns a draw `1:data$Kinf`.)

We collect all settings in the list `algo`.

```
> ## settings
> algo <- list(x0 = x0,
       neighbour = neighbour,
              nS = 5000L,
     printDetail = FALSE,
        printBar = FALSE)
```

It remains to run the algorithm.

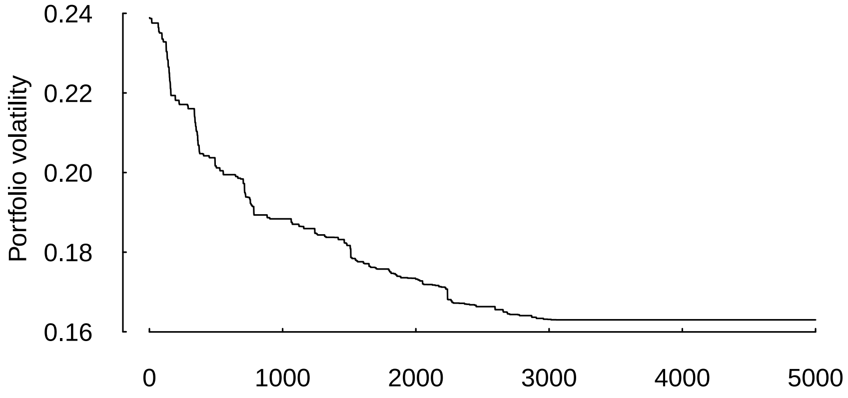
```
> system.time(sol1 <- LSopt(OF, algo, data))
```

```
   user  system elapsed
  0.556   0.000   0.556
```

```
> sqrt(sol1$OFvalue)
```

```
      [,1]
[1,] 0.163
```

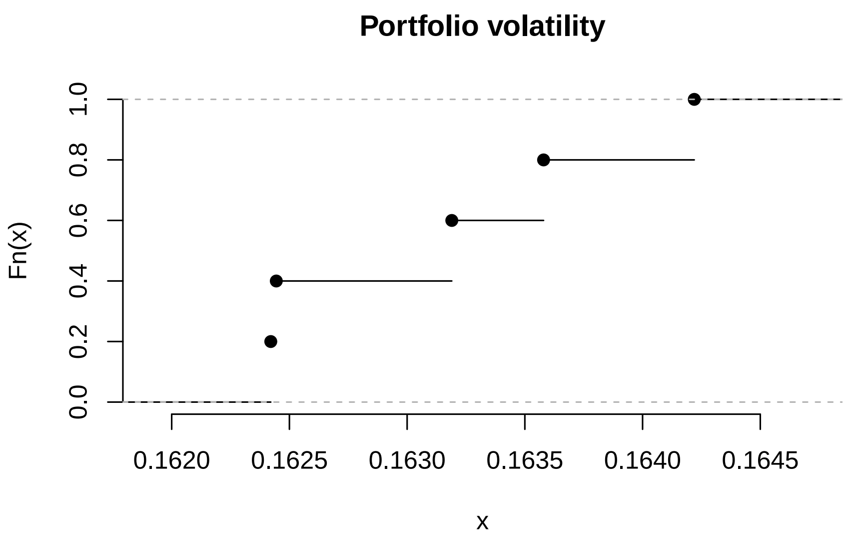
```
> par(ylog = TRUE, bty = "n", las = 1, tck = 0.01)
> plot(sqrt(sol1$Fmat[,2L]),
      type = "l", xlab = "", ylab = "Portfolio volatility")
```



(Recall that the simulated data had volatilities between 20 and 40%.)

We can also run the search repeatedly with the same starting value.

```
> nRuns <- 5L
> allRes <- restartOpt(LSopt, n = nRuns, OF, algo = algo, data = data)
> allResOF <- numeric(nRuns)
> for (i in seq_len(nRuns))
      allResOF[i] <- sqrt(allRes[[i]]$OFvalue)
> par(bty = "n")
> plot(ecdf(allResOF), main = "Portfolio volatility")
```



(We run LSopt only ten times to keep the build time for the vignette acceptable. To get more meaningful results you should increase nRuns.)

References

Manfred Gilli, Dietmar Maringer, and Enrico Schumann. *Numerical Methods and Optimization in Finance*. Elsevier, 2011.