Introduction to pair trading -Based on cointegration-

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Topics

1. What is pair trading?
2. What is cointegration?
3. Idea of pair trading based on cointegration
4. Simulation by R language
5. Summary & concluding remarks

1. What is pair trading?

Pair trading was pioneered by …

- Gerry Bamberger and Nunzio Tartaglia
- Quantitative group at Morgan Stanley
- Around 1980s
- D.E. Shaw & Co. is famous for this strategy

Pair trading is …

Market neutral trading strategy

Pair trading belongs to …

Physics, Information theory

Statistical Arbitrage

PCA, ICA, Autoregression

Neural Net

Pattern Recognition

Basic idea of pair trading …

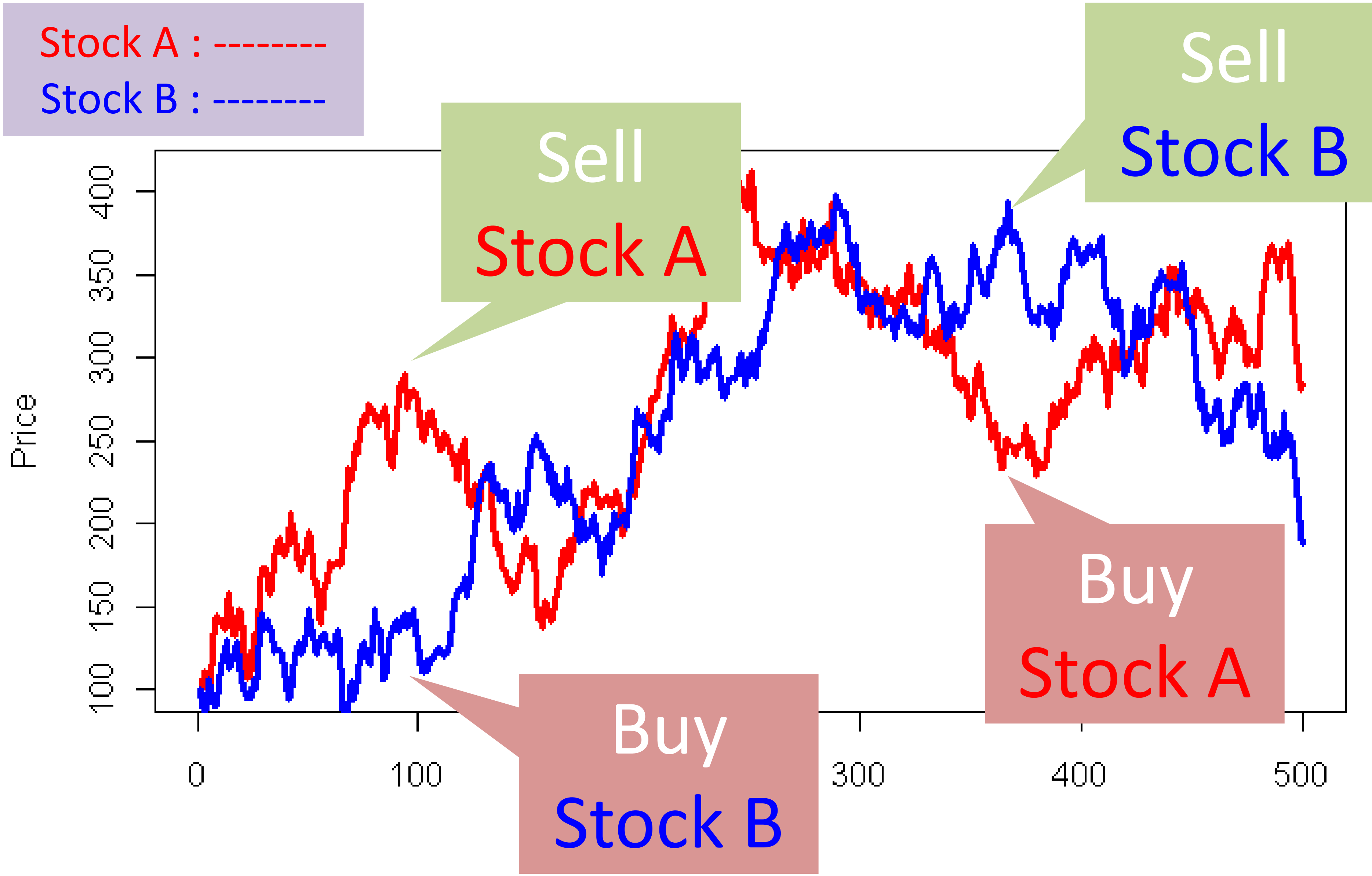
Select two stocks which move similarly

Basic idea of pair trading …

Sell high priced stock

Buy low priced stock

Basic idea of pair trading …



Basic idea of pair trading …

Usually, monitor

the difference

between two stock prices

Basic idea of pair trading …

the difference between two stock prices

2. What is cointegration?

Cointegration is …

- Pioneered by Engle and Granger
- Statistical property of time series
- Around 1990s

Cointegration is …

Not correlation

Cointegration and correlation

- **Correlation**
  - Specify co-movement of **return**
  - **Short term** relationship
- **Cointegration**
  - Specify co-movement of **price**
  - **Long term** relationship

(weak) Stationary time series

Not depend on time

- $E(X_t) = \mu$
- $\mathrm{var}(X_t) = \sigma^2$
- $\mathrm{cov}(X_t, X_{t-s}) = \gamma(s)$

Example of stationary time series

White noise

- $E(\varepsilon_t) = 0$
- $\mathrm{var}(\varepsilon_t) = \sigma^2$
- $\mathrm{cov}(\varepsilon_t, \varepsilon_s) = 0, t \neq s$

Non stationary time series

Depend on time

- $E(X_t) = \mu_t$
- $\operatorname{var}(X_t) = \sigma_t^2$
- $\operatorname{cov}(X_t, X_{t-s}) = \gamma(t, s)$

Example of non stationary time series

Brownian motion

- $E(W_t) = 0$
- $\mathrm{var}(W_t) = t$
- $\mathrm{cov}(W_t, W_{t-s}) = t - s$

Lag operator L

- $LX_t = X_{t-1}$
- $(1-L)X_t = X_t - X_{t-1} = \Delta X_t$

Integrated of order P

$$X_t : \text{non stationary}$$

$$(1-L)^p X_t : \text{stationary}$$

$$\Downarrow$$

$$X_t \sim I(p)$$

Example of “integrate”

$$Z_t = Z_{t-1} + \varepsilon_t : \text{Random walk}$$

$$\varepsilon_\mathrm{t} : \text{White noise}$$

Calculate difference

$$\Delta Z_t = Z_t - Z_{t-1} = \varepsilon_t : \text{Stationary}$$

$$\therefore Z_\mathrm{t} \sim I(1)$$

X_t and Y_t are cointegrated if …

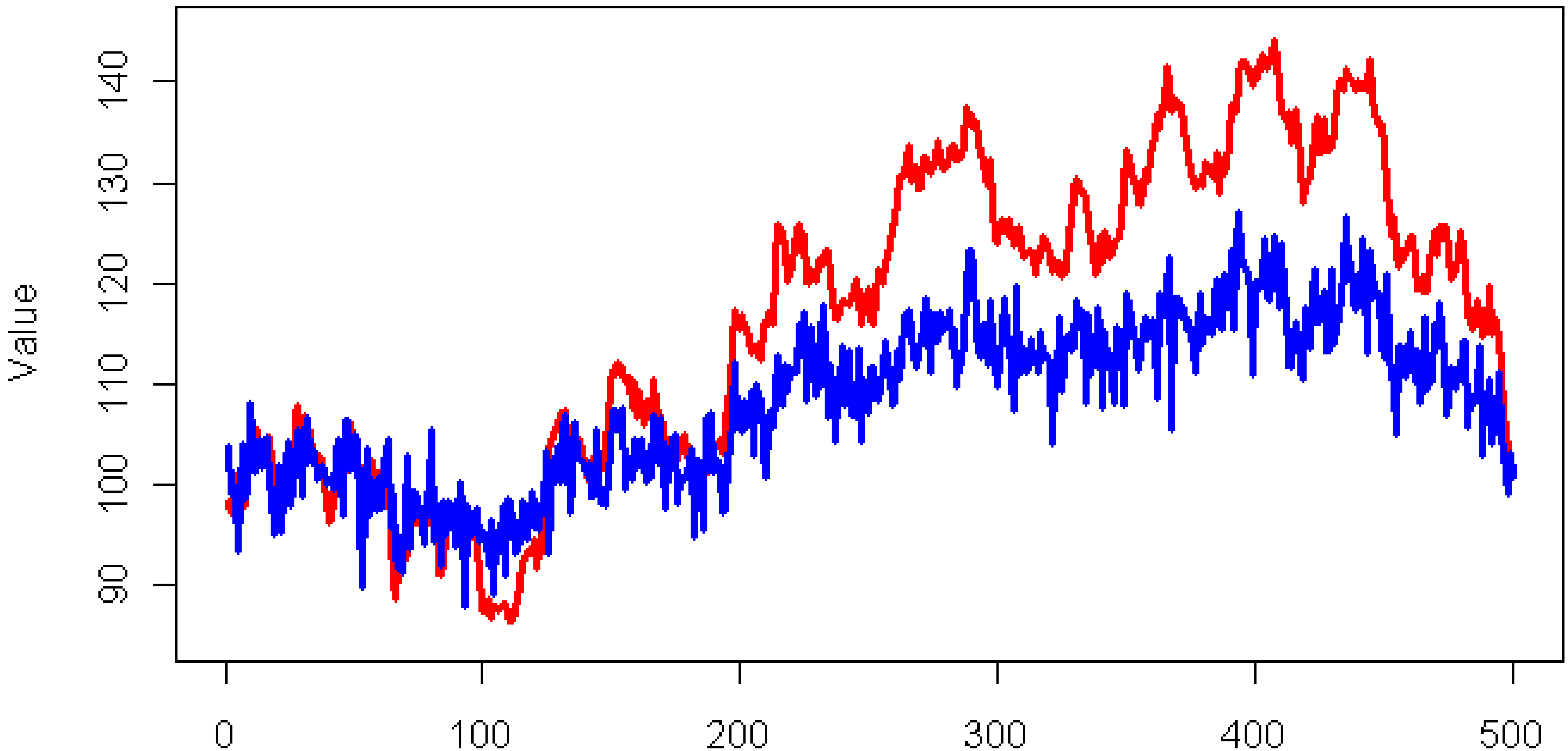
$$u_t = Y_t - \left(\alpha + \beta X_t\right)$$

$$u_t :\sim I(0), \boxed{\text{stationary process}}$$

$$X_t, Y_t :\sim I(1)$$

*This is a special version of general cointegration for I(1)

Example of cointegrated time series



Xt : --------

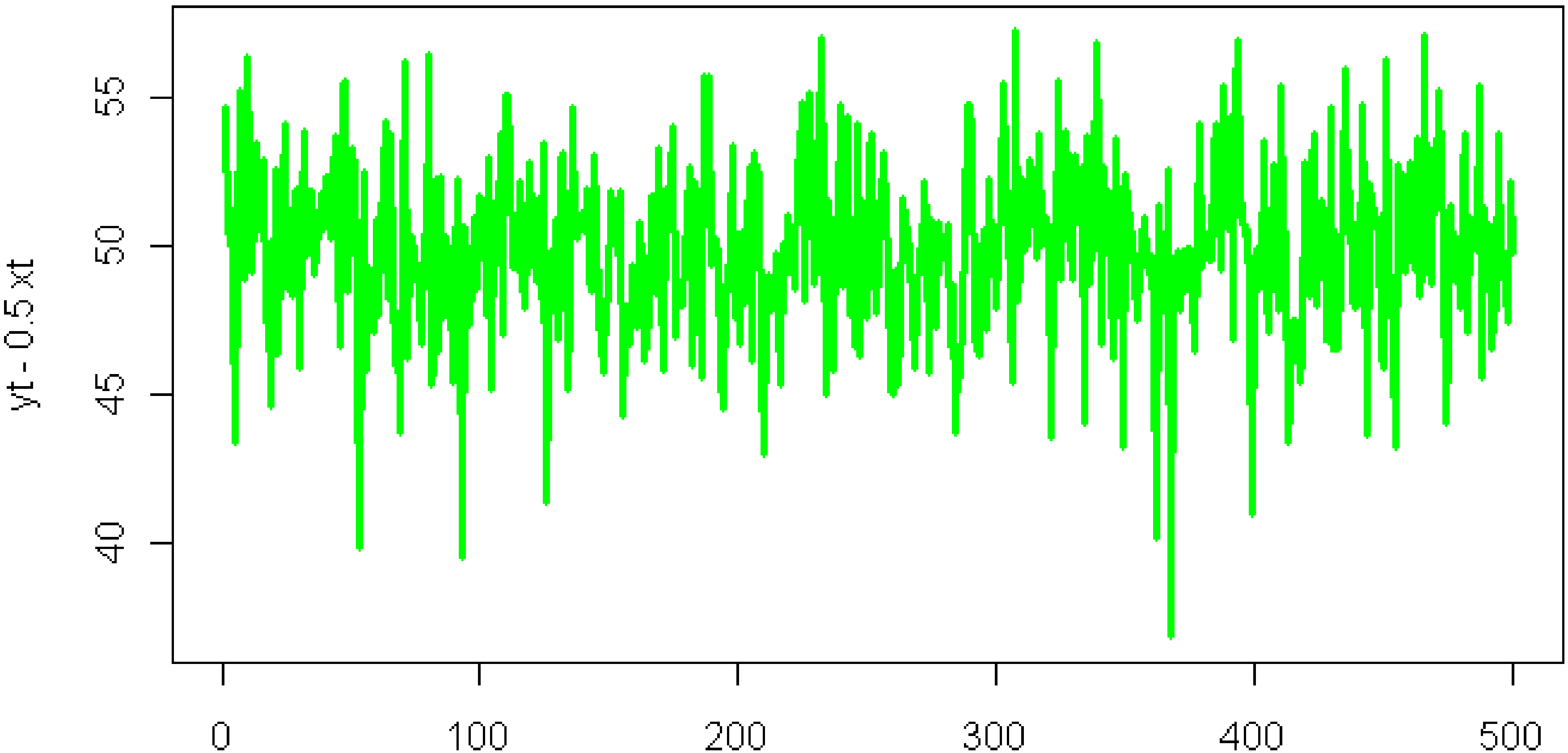
Yt : --------

$$Y_t = 50 + 0.5X_t + u_t$$

X_t : $100 + 2 \times$ Normal brownian motion

u_t : $3 \times$ Gaussian noise

Example of cointegrated time series



Plot : ut = Yt – 0.5 Xt

ut seems to be…

Stationary

&

Mean reversion

Question

Can we apply this idea to trading strategy?

3. Idea of pair trading based on cointegration

Geometric brownian motion

The most widely used model of stock price

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

S_t : Stock price

μ : Average return

σ : Volatility

W_t : Brownian motion

From Ito's lemma

$$d\log(S_t) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW_t$$

Log price follow Brownian motion

Brownian motion(log price) is …

I(1)

* Random walk can be considered as discretization of Brownian motion

Then, we can apply

Cointegration idea to log stock price

Log price spread(*) is…

Stationary

&

Mean reversion

※$Spread_t := \log(Y_t) - (\alpha + \beta \log(X_t)), \quad X_t, Y_t : \text{stock price}$

Simple trading idea

$$\text{if } Spread_t > \text{very hish} : \text{Buy } X_{t,} \text{Sell } Y_t$$

$$\text{if } Spread_t < \text{very low} : \text{Buy } Y_{t,} \text{Sell } X_t$$

$$Spread_t = \log(Y_t) - (\alpha + \beta \log(X_t))$$

$$X_t, Y_t : \text{stock price}$$

4. Simulation by R language

Process

1. Find two likely cointegrated stocks
2. Estimate spreads
3. Check stationarity
4. Create trading signal
5. Run back-test

1.Find two likely cointegrated stocks

```
> library(PairTrading)
> #load sample stock price data
> data(stock.price)
> #select 2 stocks
> price.pair <- stock.price[,1:2]["2008-12-31::"]
> head(price.pair)
           7201 7203
2009-01-05  333 3010
2009-01-06  341 3050
2009-01-07  374 3200
2009-01-08  361 3140
```

* Just load sample data in this case….

2. Estimate spreads

```
> reg <- EstimateParameters(price.pair, method = lm)
> str(reg)
List of 3
 $ spread     :An ‘xts’ object from 2008-12-30 to 2011-08-05 containing:
  Data: num [1:635, 1] -0.08544 -0.0539 -0.04306 -0.00426 -0.01966 ...
 - attr(*, "dimnames")=List of 2
  ..$ : NULL
  ..$ : chr "B"
  Indexed by objects of class: [Date] TZ:
  xts Attributes:
 NULL
 $ hedge.ratio: num 0.0997
 $ premium    : num 7.48
```

2. Estimate spreads

```
> plot(reg$spread, main = "Spread“)
```

$$Spread_t = \log(Y_t) - (\alpha + \beta \log(X_t)), X_t, Y_t : \text{stock price}$$

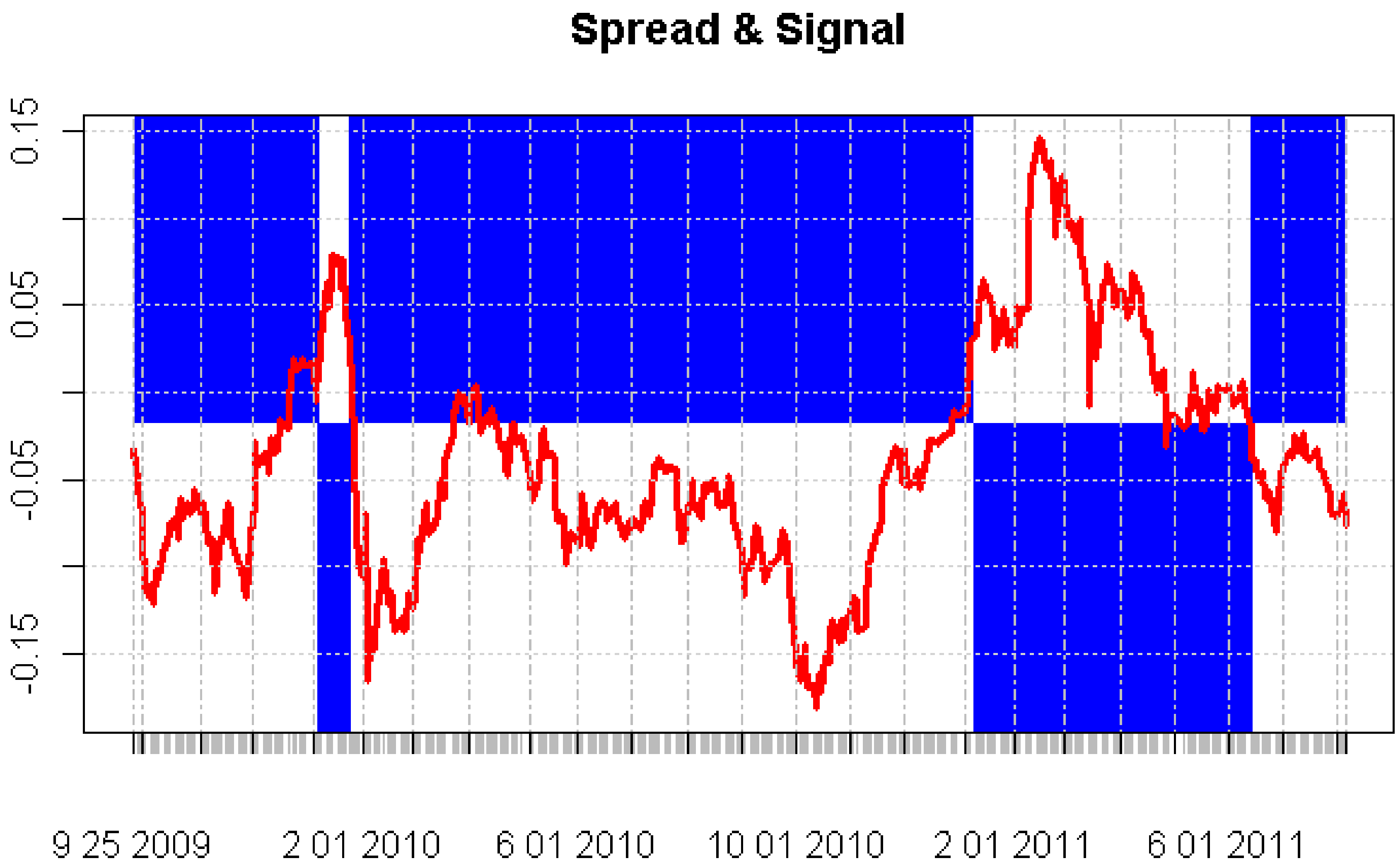
3. Check stationarity

```
> PP.test(as.numeric(reg$spread))
    Phillips-Perron Unit Root Test
data:  as.numeric(reg$spread)
Dickey-Fuller = -3.2299, Truncation lag parameter = 6, p-value
    = 0.08278
> adf.test(as.numeric(reg$spread))
    Augmented Dickey-Fuller Test
data:  as.numeric(reg$spread)
Dickey-Fuller = -3.6462, Lag order = 8, p-value = 0.02825
alternative hypothesis: stationary
```

4. Create trading signal

```
> params <- EstimateParametersHistorically(price.pair, period = 180)
> signal <- Simple(params$spread, 0.05)
> barplot(signal,col="blue",space = 0, border = "blue",xaxt="n",yaxt="n",xlab="",ylab="")
> par(new=TRUE)
> plot(params$spread, type="l", col = "red", lwd = 3, main = "Spread & Signal")
```

4. Create trading signal



5. Run back-test

```
> return.pairtrading <-
  Return(price.pair, lag(signal),
  lag(params$hedge.ratio))
> plot(100 * cumprod(1 +
  return.pairtrading), main =
  "Performance of pair trading")
```

5. Run back-test

5. Summary & concluding remarks

Summary & concluding remarks

- Pair trading is simple quantitative trading strategy
- Cointegration is long term relation ship of time series
- Idea of cointegration may give a chance to make a profit from financial market by pair trading

- Next step ….
  - Sophisticate parameter estimation & trading rule
  - Make a simulation close to real

Reference

- Pairs trade(http://en.wikipedia.org/wiki/Pairs_trade)
- Cointegration(http://en.wikipedia.org/wiki/Cointegration)
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- Russell Wojcik, “Pairs Trading: A Professional Approach”
- Daniel Herlemont, “Pairs trading, convergence trading, cointegration”
- Paul Teetor, “Using R to Test Pairs of Securities for Cointegration”(http://quanttrader.info/public/testForCoint.html)
- Ganapathy Vidyamurthy, “Pairs Trading: Quantitative Methods and Analysis “